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Cosmology and particle physics

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The cosmic connections between physics on the very largest and very smallest scales are reviewed with an emphasis on the symbiotic relation between elementary particle physics and cosmology. After a review of the early Universe as a cosmic accelerator, various cosmological and astrophysical constraints on models of particle physics are outlined. To illustrate this approach to particle physics via cosmology, reference is made to several areas of current research: baryon non-conservation and baryon asymmetry; free quarks, heavy hadrons and other exotic relics; primordial nucleosynthesis and neutrino masses.

1. Introduction

The last few years have witnessed the explosive development of a new approach to elementary particle physics via astrophysics and cosmology. Although the escalating expense of doing experimental particle physics at high energy accelerators has played some role in this trend, it is not astronomical costs alone that have driven particle physicists and astrophysicists closer together. Rather, the cosmic accelerator provided by the early evolution of the Universe offers, albeit indirectly, a site for testing the exciting predictions of grand and super-unified theories whose aesthetic successes are, as yet, unmatched by much hard data. The symbiotic relation between two such disparate fields has been a very healthy phenomenon, stimulating new and exciting work in both areas and spawning mutually beneficial and productive, cross-disciplinary collaborations. A sign of these new interactions is that while astrophysicists are striving hard to reduce uncertainties to less than a factor two, particle physicists are moving the uncertainties in their speculations into the exponent!

Why is the early Universe such a good laboratory for particle physics? Quite simply, the reason is that the early Universe was very hot and very dense. The high density and high temperature achieved during the early evolution of the Universe provided just the sort of environment experimentalists try to create when they shoot high energy beams of particles at targets or at other beams. Budgetary restrictions, however, did not limit the energies and 'currents' attained in the early Universe. In addition, and of crucial importance, the high temperatures and densities are maintained much longer in the early Universe than would be possible in collisions in accelerators.

At such high temperatures and densities collisions were very frequent and very energetic (every experimentalist's dream!). Given the long time (relatively) for which the Universe stayed at high temperatures and densities, such collisions ensured that all particles (those we already know of from laboratory experiments, as well as those not yet discovered because their masses are too high or their interactions too weak to have been produced at conventional accelerators, or both) were copiously produced by the cosmic accelerator. As the Universe expands and cools

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some of the particles produced during early epochs will survive to influence the subsequent evolution of the Universe. It is our task as scientific sleuths to search for the clues left behind – we must find the footprints of the relics. There are several ways to undertake such a search.

One such approach is to search directly for the relic particles. This option, of course, is open to us only for stable ($\tau \gtrsim t_0$ in the range 10–20 billion years), relatively strongly interacting (or electrically or magnetically charged) particles. Stable, but weakly interacting, particles (e.g. massive neutrinos) are difficult to detect directly but would contribute to the overall universal mass density; limits to the present mass density constrain the properties of such particles. Finally, primordial nucleosynthesis provides a powerful (indeed, the only) probe of the early evolution of the Universe. These approaches to particle physics via cosmology are described and illustrated. For further details and a more complete bibliography, the reader is referred to the author's recent review article (Steigman 1979).

2. RELICS FROM THE EARLY UNIVERSE

At high temperatures ($T > m$), all particles, save those very weakly interacting, will be produced copiously and will, through the processes of decays, inverse decays and scatterings, quickly come into thermodynamic equilibrium. As the Universe cools and expands, the reaction rates will fail to keep up with the universal expansion rate and there will come a time when equilibrium can no longer be maintained. At various stages then, depending on masses and interaction strengths, different particles will decouple with a 'frozen-in' surviving abundance; if stable, such relics may influence the subsequent evolution of the Universe.

It is quite simple to understand how the relative abundances of different relics depend on their properties. Consider an arbitrary but specific comoving volume V. With the exception of non-adiabatic stages in the evolution when photon entropy is generated, the volume varies with the temperature as $V \propto T^{-3}$. The number of particles in V of any specific type is $N = nV$, where n is the number density (expressed, for example, in cm^{-3}). First let us concentrate on the equilibrium situation.

For extremely relativistic (e.r.) particles ($m \ll T$), the only scale is the temperature, so that $n^{\text{e.r.}} \propto T^3$, and

$$N_{\text{eq}}^{\text{e.r.}} = n_{\text{eq}}^{\text{e.r.}}\, V \propto T^3\, T^{-3} = \text{constant}. \tag{1}$$

Thus, in equilibrium, extremely relativistic particles are conserved (i.e. the number in a comoving volume is constant). For massive particles whose interactions are sufficiently strong to be capable of maintaining equilibrium when $T < m$, the (non-relativistic, n.r.) equilibrium density falls exponentially,

$$n_{\text{eq}}^{\text{n.r.}} \propto T^{\frac{3}{2}} \exp\,(-m/T), \tag{2a}$$

and the number in a comoving volume decreases rapidly:

$$N_{\text{eq}}^{\text{n.r.}} = n_{\text{eq}}^{\text{n.r.}}\, V \propto (m/T)^{\frac{3}{2}} \exp\,(-m/T). \tag{2b}$$

Very weakly interacting particles may decouple when they are e.r.; massless particles are always e.r. In either case, the number of e.r. particles in a comoving volume is conserved since no new ones are created and no old ones are annihilated. Thus, with allowance for entropy creation,

$$N_{\text{non-eq}}^{\text{e.r.}} \approx N_{\text{eq}}^{\text{e.r.}} \approx N_\gamma, \tag{3}$$

where N_γ is the number of black-body photons in the comoving volume.

More strongly interacting, massive particles may be kept in equilibrium even when they are n.r. The number of such particles in a comoving volume decreases exponentially (see equation (2)). But, very rapidly, the very low density ensures that the relevant interactions will fail to maintain equilibrium and these, n.r., particles will decouple. The stronger the coupling, the longer equilibrium will be maintained and the lower the abundance of the surviving particles. However, because of the exponential decrease in the density, the 'freeze-out' temperature, T_*, depends only logarithmically on the cross sections (see Steigman 1979):

$$m/T_* \approx 45 + \ln(\tilde{\beta}m). \tag{4}$$

In (4), the mass is expressed in gigaelectron volts and $\tilde{\beta}$ is the thermally averaged product of the cross section and the velocity expressed in $10^{-15}\,\mathrm{cm^3\,s^{-1}}$ (for slightly non-relativistic nucleons the annihilation rate coefficient is $\beta_{\mathrm{N\bar{N}}} \equiv \langle \sigma v\rangle \approx 10^{-15}\,\mathrm{cm^3\,s^{-1}}$). Once these n.r. particles decouple, they too are conserved: no new ones are created and none annihilated. The number of particles in a comoving volume is constant and, up to factors of order 1–100 to account for entropy production, their abundance relative to black-body photons is (Steigman 1979)

$$N/N_\gamma \approx 10^{-18}(\tilde{\beta}m)^{-1}. \tag{5}$$

It is clear from equation (5) and follows from the above discussion that the lightest or most weakly interacting relics, or both, will be the most abundant. Crudely, neutral leptons will be more abundant than charged leptons which will be more abundant than hadrons.

3. Relic nucleons and baryon asymmetry

Suppose that the Universe were symmetric between nucleons and antinucleons and that baryon number were conserved. As recently as about five years ago, these assumptions would have been considered the most 'natural'. Were that the case, the results of the previous section apply (with $\tilde{\beta} \approx m \approx 1$) and predict at present a universal ratio of nucleons to photons given by

$$(N_{\mathrm{N}}/N_\gamma)_{\mathrm{symm}} \lesssim 10^{-18}. \tag{6}$$

Current estimates (see, for example, Olive *et al.* 1981) suggest that the true ratio of nucleons to photons is larger than the upper limit in equation (6) by some nine to ten orders of magnitude. This annihilation catastrophe (that only one in one to ten billion of the observed nucleons should have survived annihilation) provides strong support for our Universe being asymmetric between baryons and antibaryons (Steigman 1976). Indeed, the overwhelming weight of the relevant observational data and theoretical arguments is consistent with such an asymmetric Universe (Steigman 1976).

In contrast with the prevailing view of a few years ago, the aesthetic success of grand unified theories has convinced most of us that baryon number is not conserved. During the very early evolution of the Universe it is possible that baryon-number-violating processes could have given rise to the observed baryon asymmetry (Yoshimura 1978; Dimopoulos & Susskind 1978; Ellis *et al.* 1979; Toussaint *et al.* 1979; Weinberg 1979). Detailed calculations of the generation of baryon asymmetry (Kolb & Wolfram 1980; Fry *et al.* 1980) have shown that it is not impossible, within the content of specific GUTs and the standard big-bang model, to obtain the observed asymmetry:

$$N_{\bar{\mathrm{N}}} \ll N_{\mathrm{N}} = \eta N_\gamma, \tag{7a}$$

$$10^{-10} \lesssim \eta = N_{\mathrm{N}}/N_\gamma \lesssim 10^{-9}. \tag{7b}$$

This interplay of particle physics and cosmology has taught us an important lesson. If baryon conservation were a good symmetry we should have expected there to be an associated zero-mass-gauge boson (e.g. charge conservation $\Leftrightarrow$ photon) and, therefore, a long-range strong force. Of course, if baryon number is conserved, the proton is absolutely stable, and, if the Universe started symmetric, it would remain symmetric. We no longer believe baryon number is conserved and, thus, the proton need not be stable, and the Universe need not be (and, in general, will not be) symmetric.

In what follows, it will be convenient to compare abundances of relic particles with those of nucleons rather than those of photons (see equation (5)). In that case

$$N/N_{\mathrm{N}} = \eta^{-1}(N/N_{\gamma}) \gtrsim 10^{-9}(\tilde{\beta}m)^{-1}. \tag{8}$$

4. QUARK–HADRON TRANSITION

In estimating the present abundance of exotic relics, a distinction must be made between those that freeze out before or after the quark–hadron transition. We, therefore, should estimate the temperature, T_{c}, at which confinement occurs. This has been done crudely by Wagoner & Steigman (1979) and in more detail by Olive (1981); the following is a summary of the Wagoner–Steigman analysis.

Recall that, as the Universe expands and cools, the number density varies as $n \propto T^3$ so that the average interparticle (e.g. interquark) separation changes as $r(T) = \langle r \rangle \propto T^{-1}$. At low temperatures, particles are far apart and, for $r \gtrsim r_{\pi}$ (the 'size' of a pion), we have an ideal gas of hadrons (and leptons and photons). This situation obtains when $T < T_1 \approx 0.2\,\mathrm{GeV}$. At high temperatures we are in the quark–gluon phase. In this phase a typical quark has kinetic energy of order T and 'feels' a nearest neighbour potential $U_{\mathrm{q\bar{q}}}(r(T))$. As T decreases $\langle r \rangle$ increases and $U_{\mathrm{q\bar{q}}}(\langle r \rangle)$ increases. Colour forces will begin to dominate when $T \approx U_{\mathrm{q\bar{q}}}(r(T))$; this occurs for $T_2 \approx 0.4\,\mathrm{GeV}$. Thus, for low temperatures ($T < T_1$) we have an ideal gas of hadrons, and for high temperatures ($T > T_2$) an ideal gas of quarks and gluons. In between, there is a transition from quarks and gluons to hadrons. The temperature at which confinement occurs, T_{c}, lies between T_1 and T_2. Recall that for strongly interacting particles, freeze-out occurs at $T_* \approx m/45$ so that

$$T_*/T_{\mathrm{c}} \approx m/10\,\mathrm{GeV}. \tag{9}$$

Thus, heavy, strongly interacting particles will freeze out before confinement ($T_* > T_{\mathrm{c}}$).

5. FREE RELIC QUARKS

It is the establishment view that colour is confined and, therefore, that coloured quarks can never be free. If this is true, then there can be no free quarks surviving as relics from the big bang. If, however, the experiments of La Rue *et al.* (1977, 1979*a*, *b*) are correct and are to be interpreted as evidence for free quarks, then colour is not confined and free relic quarks should exist.

The first estimates of the abundance of relic quarks were made by Zeldovich *et al.* (1965, 1966) who considered quark production and annihilation in collisions with hadrons. In this case, it follows from equation (8) that

$$N_{\mathrm{q}}/N_{\mathrm{N}} \gtrsim 10^{-9}(\tilde{\beta}_{\mathrm{q\bar{q}}}\, m_{\mathrm{q}})^{-1}. \tag{10}$$

Note, however, the presence of hadrons requires that $T_* \lesssim T_{\mathrm{c}}$ so that $m_{\mathrm{q}} \lesssim 10\,\mathrm{GeV}$. In this case, $N_{\mathrm{q}}/N_{\mathrm{N}} \gtrsim 10^{-12}$. According to this estimate, relic quarks would be all too abundant. As Wagoner

& Steigman (1979) have noted, however, the estimate of Zeldovich *et al.* (1965, 1966) does not apply if $m \gtrsim 10\,\mathrm{GeV}$ so that $T_* \gtrsim T_c$.

Consider then the case where $m \gtrsim 10\,\mathrm{GeV}$. For $T > T_c$ all quarks are unconfined. When the temperature drops to T_c, most quarks are confined, *but* there will be some quarks, in the tail of the distribution, with such high energy that $E_q > m_q$. Those exponentially rare quarks will remain free. Wagoner & Steigman (1979) have estimated the abundance of such quarks as

$$N_q/N_N \approx 10^9\,(m_q/T_c)^2 \exp\,(-m_q/T_c). \tag{11}$$

For $15 \lesssim M_q \lesssim 30\,\mathrm{GeV}$ the relic quark abundance is similar to that inferred from the Stanford experiments.

6. Heavy quarks–exotic hadrons

Suppose there are 'new' heavy quarks, the lightest ($m_Q > 10\,\mathrm{GeV}$) of which is stable (e.g. sextet quarks). Such quarks would freeze out before confinement ($T_* > T_c$) with a relatively high abundance (Dover *et al.* 1979):

$$f_* = N_Q/N_N \approx 10^{-6}\,(m_Q/m_N) \gtrsim 10^{-5}. \tag{12}$$

At confinement these new quarks may combine with the usual colour triplet quarks to form exotic heavy hadrons (H). Compared to the point-like quarks, the heavy hadrons are large ($\sigma_{H\bar{H}} \gg \sigma_{Q\bar{Q}}$) so that at confinement there will be renewed annihilation. Since $T_c \ll m_H$ there will be no new production. The present abundance of such exotic relics is, therefore, much below their abundance at freeze-out (Dover *et al.* 1979):

$$f = N_Q/N_N \approx 10^{-10}. \quad (13$$

Notice that if the same (or similar) mechanism that is responsible for the observed baryon asymmetry had operated for these new quarks, they might have been as abundant as nucleons. In summary, then, stable ($\tau > t_0$) exotic relics might be expected with an abundance f that lies in the range $10^{-10} - 1$. Dover *et al.* (1979) and, more recently, Ellis *et al.* (1981) have considered the constraints on such relics. These are summarized below; for details and further references see Ellis *et al.* (1981).

(*a*) The absence in accelerators of new strongly interacting, stable particles suggests

$$m \gtrsim 10\,\mathrm{GeV}\ (T_* \gtrsim T_c).$$

(*b*) For stable relics ($\tau > t_0$) there would be too much mass in the Universe ($\rho \gtrsim 10\rho_N$) unless $f \lesssim 10\,m_N/m$.

(*c*) To avoid discrepancies between physical and chemical mass determinations, the abundances of anomalous isotopes must be small: $f \lesssim 10^{-4}\,(m_N/m)$.

(*d*) Directed searches for specific anomalous nuclei (Middleton *et al.* 1979) suggest $m \gtrsim 60\,m_N$.

(*e*) Symmetric relics that stopped annihilating in the early Universe might, unless shielded from each other, renew annihilation in the high density environment of galaxies. The γ-ray background requires that $f \lesssim 10^{-7}\,\beta_\gamma^{-1}$ where β_γ is the number of γ-rays per annihilation.

(*f*) In analogy with the predicted nucleon decay, exotic hadrons might not be absolutely stable. If β_μ is the number of muons produced in the decay of a heavy hadron, current limits (Crouch *et al.* 1978) require $\tau \gtrsim 5 \times 10^{30} f(m/m_N)\,\beta_\mu$ a.

(*g*) Short-lived exotics ($\tau < t_0$) are more difficult to constrain. For $\tau \gtrsim 10^7$ a, the γ-ray observations exclude relics more abundant than $f \approx 10^{-8}$. Shorter-lived relics may have avoided distorting the microwave background or producing too much entropy, or both, provided that they decay quickly enough: $\tau \lesssim 10^{6}\,(fm/m_{\mathrm{N}})^{-2}$ a. Finally, very short-lived exotics that decayed before nucleosynthesis ($\tau \lesssim 10^{-8}$ a) could not be constrained.

7. Nucleons, nucleosynthesis and relic neutrinos

I conclude with one final illustration of the approach to particle physics via cosmology. We consider the possibility that relic neutrinos, with a small but finite rest mass, dominate the density of the Universe. First, let us consider nucleons and the universal mass density.

The present expansion rate of the Universe is measured by the Hubble parameter, H_0, which unfortunately remains uncertain. To parametrize the uncertainty, introduce h_0 such that

$$H_0 = 100\,h_0\,\mathrm{km\,s^{-1}\,Mpc^{-1}}; \quad \tfrac{1}{2} \leqslant h_0 \leqslant 1. \tag{14}$$

The Hubble age, H_0^{-1}, is related to the present age of the Universe: $t_0 \leqslant H_0^{-1} \approx 9.8\,h_0^{-1} \times 10^{9}$ a. To H_0 there corresponds a critical density

$$\rho_{\mathrm{c}} = 3H_0^2/8\pi G \approx 2\times10^{-29}\,h_0^2\,\mathrm{g\,cm^{-3}}. \tag{15}$$

It is convenient to compare all densities with ρ_{c} since for $\rho_0 > \rho_{\mathrm{c}}$ the Universe is closed and the expansion will eventually stop, to be followed by collapse, whereas for $\rho_0 < \rho_{\mathrm{c}}$ the Universe is open and will expand forever. The density parameter, Ω_0, is just the ratio of the present density ρ_0 to ρ_{c}. Now let us turn to the nucleon density.

It is convenient to compare all number densities to the density of black-body photons.

$$n_{\mathrm{N}} = \eta n_{\gamma}; \quad n_{\gamma} = 400\,(T_0/2.7)^{3}\,\mathrm{cm^{-3}}. \tag{16}$$

In terms of the nucleon:photon ratio η, the nucleon contribution to the total density is

$$\Omega_{\mathrm{N}} = (0.0035/h_0^2)\,\eta_{10}\,(T_0/2.7)^{3} \lesssim 0.02\eta_{10}, \tag{17}$$

where $\eta_{10} = 10^{10}\eta$ and $T_0 \lesssim 3.0$ K.

We may now use the results of primordial nucleosynthesis to constrain η (see, for example, Schramm & Steigman 1980, 1981). If the primordial abundance by mass of ^{4}He is no more than 0.25, and if there are at least three, two-component neutrinos then $\eta_{10} \lesssim 5$. Nucleosynthesis, then, argues for a low-nucleon-density Universe: $\Omega_{\mathrm{N}} \lesssim 0.1$. There is some evidence that $\Omega_0 \gtrsim 0.2$, suggesting that the Universe is not nucleon-dominated (Schramm & Steigman 1980, 1981). Furthermore, it is very difficult to form galaxies in such a low-density Universe, which implies again that something else may dominate the density. What about massive relic neutrinos?

The usual, neutral current-weak interactions will keep 'light' ($m_{\nu} \ll 1$ MeV) neutrinos in equilibrium at temperatures greater than about 1 MeV:

$$\mathrm{e^{+}+e^{-}} \leftrightarrow \nu_{\mathrm{i}}+\bar{\nu}_{\mathrm{i}} \quad (\mathrm{i} = \mathrm{e}, \mu, \tau, \ldots). \tag{18}$$

In equilibrium, $T_{\nu} = T_{\mathrm{e}} = T_{\gamma}$ and the ratio of neutrinos to photons is

$$n_{\nu}/n_{\gamma} = \tfrac{3}{4}(g_{\nu}/g_{\gamma}) = \tfrac{3}{4}(g_{\nu}/2), \tag{19}$$

where g_{ν} is the number of neutrino helicity states. Below $T \approx 1$ MeV, the neutrinos freeze out. When e$^{\pm}$ pairs later annihilate ($T_{\gamma} < m_{\mathrm{e}}$) their energy is transferred to the photons, but not to the decoupled neutrinos:

$$(T_{\nu}/T_{\gamma}) = (\tfrac{4}{11})^{\frac{1}{3}}, \quad n_{\nu}/n_{\gamma} = \tfrac{3}{11}(g_{\nu}/2). \tag{20}$$

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Thus, today, in every cubic centimetre there are about 400 microwave photons and a comparable number of 'microwave neutrinos'. If the neutrinos have a non-zero rest mass, the ratio of mass in neutrinos to mass in nucleons is (with m_ν in electronvolts)

$$\frac{\Omega_\nu}{\Omega_{\mathrm{N}}} = \frac{3m_\nu}{\eta_{10}}; \; m_\nu \equiv \sum_i (g_{\nu i}/2)\, m_{\nu i}. \tag{21}$$

If, indeed, $\eta_{10} \lesssim 5$ then neutrinos will dominate over nucleons if $m_\nu \gtrsim 1.7$ eV. A neutrino mass in excess of a few electronvolts would ensure that we live in a neutrino-dominated Universe. The neutrino contribution to the total density is

$$\Omega_\nu = [m_\nu/100h_0^2]\,(T_0/2.7)^3. \tag{22}$$

Note that, even if m_ν were known, the uncertainty in h_0 (and, to a lesser extent, in T_0) would be reflected in an uncertainty in Ω_ν.

We may use a limit to the age of the Universe to constrain m_ν (Gershtein & Zeldovich 1966). The age of the Universe is less than the Hubble age (H_0^{-1}) by an amount that depends on Ω_0. If we insist that the Universe be at least ten billion years old we are led to the constraint (Schramm & Steigman 1980, 1981)

$$\Omega_\nu h_0^2 \lesssim 1, \quad m_\nu \lesssim 100\,\mathrm{eV}. \tag{23}$$

From cosmology then, we learn that our Universe will be neutrino-dominated if the sum of the neutrino masses exceeds a few electronvolts. Furthermore, the sum of the masses of stable ($\tau \gtrsim t_0$) neutrinos cannot exceed about 100 eV.

8. Summary

In the last few years we have witnessed the birth and growth to healthy adolescence of a new collaboration between astrophysicists and particle physicists. The most notable success of this cooperative effort has been to provide the framework for understanding, within the context of GUTs and the hot big-bang cosmology, the universal baryon asymmetry. The most exciting new predictions this effort has spawned are that exotic relics may exist in detectable abundances. In particular, we may live in a neutrino-dominated Universe. In the next few years, accumulating laboratory data (for example proton decay, neutrino masses and oscillations) coupled with theoretical work in particle physics and cosmology will ensure the growth to maturity of this joint effort.

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